Among its many provisions, No Child Left Behind (NCLB) requires that states adopt “challenging academic content standards” in mathematics, reading/language arts, and science. These standards must specify what children are expected to know and be able to do, must contain coherent and rigorous content, and must encourage the teaching of advanced skills. What’s more, the states are required to measure the achievement of students against the state standards in grades 3 through 8. Since 2002, 38 states have developed or revised their mathematics curriculum standards, some of which are intended to serve as “models” for local districts, while others are mandatory and specify the mathematics all students in the state are expected to learn at particular grades. The state curriculum standards serve as guidelines for shaping each state’s annual grade-level assessments.

All of that is old news to most Kappan readers. What might be new news is that the new state standards do not reflect anything like a consensus of opinion regarding when

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Consensus or Confusion?

The Intended Math Curriculum in State-Level Standards

While most states have developed well-articulated mathematics standards, including specific grade-level expectations, Ms. Reys and Ms. Lappan found no consensus — and the potential for much confusion — when they conducted a national study of state mathematics standards.

By Barbara Reys and Glenda Lappan
students should learn particular topics in mathematics. In fact, the variation in grade placement of topics across the state standards is quite likely to contribute to the continued development of textbooks that are repetitive and provide superficial treatment of a range of topics.

AUTHORITY OF STANDARDS DOCUMENTS

The newest iterations of state mathematics standards specify grade-level learning expectations. For many states, these documents are far more specific with regard to grade placement of topics than previous state standards or frameworks. For example, prior to NCLB, most state departments of education provided school districts with a broad set of standards (generally organized by grade band, such as K-4, 5-8, and 9-12). The states then monitored student learning at particular grades (e.g., grades 4, 8, and 10). School districts were encouraged to use these broad guidelines to create more detailed learning goals for each grade. But because of the mandates of NCLB, states that had not previously provided much detail have now created grade-by-grade learning goals for mathematics. The relationship of these standards to high-stakes assessments in the states has given the documents new authority and made them far more relevant than they had been before.

According to respondents to a recent survey of state-level mathematics curriculum supervisors, most teachers and school administrators today are paying closer attention to the curriculum standards provided by state education agencies than they did in the past. In fact, more than two-thirds of respondents perceived the new state-level curriculum standards to be significantly influencing classroom instruction, textbook selection, and professional development for teachers.1

In an effort to understand the nature of the new standards and the level of consensus across states, we reviewed all state mathematics standards that outline grade-specific learning goals for at least grades 3-8 (41 states in all, plus the Department of Defense Education Agency). Findings from this study confirm that mathematics learning expectations vary across the states along several dimensions, including level of specificity, language used to convey learning goals, and grade placement of specific learning expectations. We summarize our findings on each of these dimensions below.

VARIATION ACROSS STATES

Specificity and complexity. We noted two major areas of variation in the structure of grade-level learning expectations (GLEs) within state standards. The first is the level of specificity of the GLEs. For example, the Arizona standards on “functions” include the same GLE in each of grades 4-8, using the words “grade-level appropriate” to differentiate between the specific expectations at each level. The corresponding Colorado standards include grade-specific GLEs, differentiated according to the types of numbers used in the functions (e.g., whole numbers versus rational numbers). Similarly, the Maryland standards include grade-specific GLEs related to function, but they also include specifications regarding assessment limits (i.e., guidance as to what will be included on the state-level assessment for this topic). These three examples also illustrate differences in what mathematics is expected of students in these three states. Arizona and Maryland focus on function tables and rules, while Colorado focuses on relationships and descriptions of how a change in one quantity affects the other. These may seem like minor differences, but they arguably represent quite different foci for what is expected of students.

A second area of variation noted in our analysis relates to the “grain size” and complexity of the learning expectations. For example, some state documents pack a single GLE with multiple learning goals, while other documents include GLEs with a simpler structure, focusing on one goal or idea. The GLE on understanding equality from the New Hampshire standards for eighth grade is complex in structure and includes multiple learning objectives. The corresponding Ohio standards focus on similar ideas but are organized in a series of individual GLEs. Given such differences in grain size and complexity, comparing the number of GLEs across state documents can be misleading.

Language and cognitive demand. In addition, the cognitive demand suggested by the language used in GLEs varies by state and grade level. In attempting to understand how demanding the various state standards documents are, we analyzed the words that implied student actions in the algebra GLEs included in the 42 standards documents. These tended to be the verbs in the standards statements, and we identified 99 verb families (all forms of a root verb were counted as a single verb family: apply, applying, and applied
were counted as one). The three verbs that were most frequently used in the algebra standards across the grades were use, describe, and solve. We found a near absence of verbs that called for higher levels of thinking and reasoning, such as explain, generalize, justify, or prove.

Some state standards introduce computation with fractions as early as grade 1, while others begin instruction on the topic in grade 3 or 4. Some expect students to be fluent in computing with fractions by the end of grade 5, and other state standards include this expectation at grade 8.

Table 1 summarizes the results of our review of GLEs for generalizing patterns/formulating rules based on patterns. For example, while students are expected to “describe” patterns, “analyze” patterns, “state rules for” patterns, or “generalize patterns” throughout the elementary grades, in general, the emphasis in grades 4-8 is on describing rules and understanding and applying those rules. These categories of verbs suggest levels of cognitive demand. Using Bloom’s Taxonomy as a referent, the expectations move from simply describing the rule to applying and analyzing it, to justifying one’s reasoning, to validating a rule. When considered in this way, the totals across the bottom of the table take on added significance: there are fewer expectations at the higher levels of cognitive demand. Students are asked to justify the rule for a pattern in only five GLEs. In contrast, they are asked to describe a rule in 153 GLEs.

Grade placement. Based on our review, the grade at which key mathematical topics are introduced and the trajectory of their development across grades differ dramatically from state to state. For example, the ability to compute with fractions is a core idea for the elementary and middle-grade mathematics curriculum. Some state standards introduce computation with fractions (using common fractions such as $\frac{1}{2}$) as early as grade 1, while others begin instruction on the topic in grade 3 or 4. Some state standards include an expectation for students to be fluent in computing with fractions by the end of grade 5, and other state standards include this expectation at grade 8.

Figure 1 provides a summary of the grade at which students are introduced to the addition and subtraction of fractions and when proficiency is expected, as outlined in the 42 standards documents examined. As noted, the states differ in when they introduce addition and subtraction of fractions (ranging from grade 1 to grade 7), in the number of years they develop the topic (ranging from one year to six), and in the grade level at which students are expected to be proficient in addition and subtraction of fractions (ranging from grade 4 to grade 7). The variation noted here is not unique to this one topic but is evident in all the topics we analyzed.

Similar differences in the grade placement of learning expectations are evident in the algebra strand. While few abilities related to algebra are to be mastered in the elementary and middle grades, there is ample evidence that states vary substantially in the grade levels at which they concentrate on particular topics in algebra. For example, the levels at which states expect the commutative property of multi-

<table>
<thead>
<tr>
<th>Grade Level</th>
<th>Express/Describe Rules</th>
<th>Understand and Apply Rules</th>
<th>Analyze Rules</th>
<th>Explain/Justify Rules</th>
<th>Total</th>
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<tr>
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<td>19</td>
<td>7</td>
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</tr>
<tr>
<td>Total</td>
<td>153</td>
<td>48</td>
<td>10</td>
<td>5</td>
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</table>
application to be taught vary from grade 2 to grade 8, with grades 3 and 4 having the largest concentration. The grade levels at which states expect knowledge of variables range from kindergarten to grade 8, with the greatest emphasis falling in grade 4 to grade 7.

**IMPLICATIONS OF STATE-BY-STATE VARIATION**

Given the increased influence of state-level standards, the extent to which the content emphasized at various grade levels is the same or different has implications for the development of publisher-generated textbooks and for interpreting student learning gains across the states.

As publishers create textbooks for use in schools, they pay close attention to the newest generation of state standards. If the textbooks developed are to be marketed and sold within a state, they must align with the state’s standards. That is, the topics called for in the standards must be addressed within the appropriate grade-level textbook. In order to produce a textbook that can be used in many states, publishers tend to include many more topics than any one state calls for at any one grade level, thus increasing the likelihood of large textbooks that treat many topics superficially. Another concern is that, as publishers attempt to rearrange materials into grade levels to fit specific state standards documents, the coherence of the development of important concepts and related processes will be compromised.

Given the variation in goals for student learning at particular grades, comparisons of student performance across states are rendered meaningless. Not only do states use different assessments, but they also have set different learning goals. Thus a high-performing student in one state may...
or may not have the same knowledge of mathematics as a high-performing student in another state.

RECOMMENDATIONS

As states plan for the next cycle of review, we offer the following suggestions.

• **Identify two to four major mathematical goals at each grade level, K-8.** At each grade, we recommend a general statement of major goals for the grade. These general goals may specify emphasis on a few strands of mathematics or a few topics within strands. These general goals should be coordinated across all grades, K-8, to ensure curricular coherence and comprehensiveness.

• **Limit the number of grade-level learning expectations so as to focus instruction and deepen learning.** The set of learning expectations per grade level should be manageable in the course of a school year. Along with the statement of general goals and priorities for a particular grade, we suggest that the set of learning expectations per grade be limited to 20-25. This number is similar to the number of goals in the curriculum standards documents of other countries.4

• **Collaborate to promote consensus.** Fifty states with 50 standards documents is a recipe for mammoth textbooks that treat many topics superficially. In order to increase the likelihood of focused curriculum materials, states need to work together to create some core of consensus about important learning goals and expectations at each grade. In fact, we recommend that a consortium of national groups collaborate to propose a national core curriculum that focuses on *priority* goals for each grade, K-8. In this way, states might still tailor their own curriculum goals and expectations to local needs while ensuring a much greater level of consistency across the nation.

SUMMARY

Clearly, much work and effort at the state level have gone into articulating learning expectations for mathematics in grades K-8. The state GLE documents present specific learning goals and describe learning sequences for attaining these goals across the elementary years. For many states, these grade-level learning expectations represent a new level of state leadership for curriculum articulation. Yet considered as a national collection, they highlight a consistent lack of national consensus regarding common learning expectations in mathematics at particular grade levels.

As states look ahead to the next cycle of review and revision of mathematics standards, it is likely that they will continue to look to national groups for curriculum leadership. In fact, 85% of respondents to a recent survey of state mathematics curriculum supervisors indicated that national leadership is needed to assist in the future articulation of curriculum standards in mathematics, particularly from national professional organizations of mathematics teachers (K-12 and university) and from mathematicians.4 National leadership is needed to increase the level of expertise and resources in developing a well-articulated mathematics curriculum and to promote more consensus.

We have offered some recommendations for the future development of mathematics curriculum standards documents. Our suggestions are intended to help improve the coherence, continuity, and articulation of the mathematics curriculum offered to U.S. students. It will take strong leadership, cooperation, and collaboration to realize the goal of a coherent, rigorous mathematics curriculum for all U.S. students. But there is no better time to begin this work than right now.


2. For a full report of the findings, see Barbara J. Reys, ed., *The Intended Mathematics Curriculum as Represented in State-Level Curriculum Standards: Consensus or Confusion?* (Greenwich, Conn.: Information Age Publishing, 2006).


5. Reys et al., op. cit.